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Electron–positron pair production by linearly polarized photon in the nuclear field

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Abstract

Process of lepton pair production by polarized photon on nuclei can be used to measure the degree of linear polarization of high energy photon. The differential cross section and the analyzing power are calculated with taking into account higher powers of expansion in $Z\alpha$. Pure Coulomb and screened potential are considered. The results are illustrated in terms of asymmetry. The screening effect as well as the effect of higher orders expansion in $Z\alpha$ is shown to vanish in limit of pair emission angles exceeding m/ω.

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1. Introduction

Studies of a pair creation process started in the celebrated papers of 1953–1969 continue to attract attention up to now [1–13]. Main interest nowadays is the use of this process as a polarimeter [14]. Really, it has rather a large cross section and the polarization effects can reach 14 per cent [12]. Two different mechanisms of pair creation must be taken into account: the Bethe–Heitler one when the pair is produced in collision of two photons, one real and the other virtual, and the bremsstrahlung mechanism when a pair is created by a single virtual photon. It was shown in the fundamental papers of E. Haug [13] that at photon energies exceeding 50 MeV in the target laboratory frame the contribution of the bremsstrahlung mechanism as well as the interference of the corresponding amplitude with two-photon ones does not exceed 5 per cent and decreases with further photon energy growth. Taking into account the lowest order radiative corrections (RC) does not change the situation. In the case of target such as proton or light nuclei the main contribution to (RC) is connected with a final state interaction between pair components. In the two virtual photon exchange between a particle and nuclei amplitude does not interfere with the Born amplitude as they have different signatures. The pure two-photon exchange amplitude contribution does not contain any enhancement factors such as “large logarithms” of ratio of photon energy ω to lepton mass m, and is of order α^2. It can be neglected compared to the contribution of order α/π coming from interference of the Born amplitude with the 1-loop ones connected with lepton pair interaction.

The situation changes when one consider the pair creation on heavy nuclei with the charge parameter $\nu = Z\alpha$ being not too small. The main contribution arises from a many-photon exchange mechanism between a pair component with nuclei.

The total cross section of pair creation process by photon on nuclei of charge Z,

$$\begin{aligned}&\gamma(k) + Y(P,Z) \to e^-(p_1) + e^+(p_2) + Y(P',Z), \qquad q = P - P', \qquad s = 2Pk = 2M\omega,\\ &P^2 = (P')^2 = M^2, \qquad p_\pm^2 = m^2, \qquad |q^2| \ll s,\end{aligned} \tag{1}$$

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for unpolarized photon is [7,8]

$$\sigma = \frac{28}{9}\frac{Z^2\alpha^3}{m^2}\left[\ln\frac{2\omega}{m} - \frac{109}{42} - f(\nu)\right], \quad f(\nu) = \nu^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\nu^2)}. \tag{2}$$

This result has recently been reproduced in an informative paper of Ivanov an Melnikov [9] where the differential cross section was considered as well.

The direction of e^+ and e^- emittance correlates with the degree and direction of photon linear polarization and so this process can serve as a polarimetric reaction for measuring linear polarization of high energy photons, at the present, beams in the region of photons. At the present time, experimental tasks request measurements of linear polarization of photon beams in the region of photon energy up to 1–2 GeV with an accuracy of about 1 per cent or better. So one needs to know the cross section of e^+e^- pair photo-production and the analyzing power of this appropriate accuracy.

The considered process was discussed in detail in the works mentioned above. Calculations in these works were carried out on the basis of wave functions of the final electron and positron in the external screened Coulomb field in the Furry–Sommerfeld–Maue approximation. This approximation is valid for high energy of produced particles, $m/\epsilon_{1,2} \ll 1$, and for small emitting angles $\theta_{1,2} \sim m/\epsilon_{1,2} \approx 10^{-3}$; $\epsilon_{1,2}$ is the energy of electron and positron, and m is the electron mass.

It is well known that the main contribution to the cross section of the considered process gives just the region of small emitting angles. However, it should be noted that for the purposes of high-energy photon polarimetry, the restriction on the value of angles observable in the experiment must be imposed. In this Letter, we use formalism of [9] to consider the case of linearly polarized photon.

First, we briefly sketch the relevant results of [9]. Sudakov's parametrization of 4-momenta is used below

$$q = \alpha_q k + \beta_q \tilde{P} + \vec{q}, \qquad q_i = \alpha_i k + \beta_i \tilde{P} + \vec{q}_i, \qquad p_1 = x_1 k + y_1 \tilde{P} + \vec{p}_1, \qquad p_2 = x_2 k + y_2 \tilde{P} + \vec{p}_2, \tag{3}$$

with $\vec{a}$-Euclidean two-dimensional vector $\vec{a} = (0, 0, a_x, a_y)$ orthogonal to photon 4-momentum $k = \omega(1, 1, 0, 0)$; $\tilde{P} = (M/2)(1, -1, 0, 0) = P - k(M^2/s)$ is the light-like 4-vector. The conservation law and on mass shell conditions lead to

$$x_1 + x_2 = 1, \qquad y_1 = \frac{c_1}{x_1 s}, \qquad y_2 = \frac{c_2}{x_2 s}, \qquad c_l = \vec{p}_l^{\,2} + m^2, \quad l = 1, 2; \qquad \vec{q} = \vec{p}_1 + \vec{p}_2. \tag{4}$$

The matrix element corresponding to N photon exchange is

$$M_N = -i^N s\frac{8\pi^2(eZ)^N}{N!}\int\prod_{i=1}^{N}\frac{d^2q_i}{(2\pi)^2}\frac{F(q_i^2)}{\vec{q}_i^{\,2}}\delta^{(2)}\Big(\sum q_i - q\Big)J^N_{\gamma\to l\bar{l}}, \tag{5}$$

where $J^{(N)}_{\gamma l\bar{l}}$ is the impact factor which is rewritten in the simple form [8–10] with

$$J^{(N)}_{\gamma\to l\bar{l}}(\vec{p}_1, \vec{p}_2) = \bar{u}(p_1)\big[mS^{(N)}\hat{\epsilon} - 2x_1\vec{T}^{(N)}\vec{\epsilon} - \hat{T}^{(N)}\hat{\epsilon}\big]\frac{\hat{\tilde{P}}}{s}v(p_2). \tag{6}$$

The quantities $S, \vec{T}$ obey the recurrent relations

$$S^{(N)}(\vec{p}_1, \vec{p}_2, \vec{q}_N) = S^{(N-1)}(\vec{p}_1, \vec{p}_2 - \vec{q}_N) - S^{(N-1)}(\vec{p}_1 - \vec{q}_N, \vec{p}_2), \quad N = 2, 3, \ldots, \tag{7}$$

and a similar expression for $\vec{T}^{(N)}$. The initial values, corresponding to the one-photon exchange are

$$S^{(1)} = S^{(1)}(\vec{p}_1, \vec{p}_2) = \frac{1}{c_1} - \frac{1}{c_2}; \qquad \vec{T}^{(1)} = \vec{T}^{(1)}(\vec{p}_1, \vec{p}_2) = \frac{\vec{p}_1}{c_1} + \frac{\vec{p}_2}{c_2}. \tag{8}$$

Introducing the values

$$J^{(N)}_{S,\vec{T}} = \int\prod_{i=1}^{N}\frac{d^2q_i\,F(q_i^2)}{\vec{q}_i^{\,2}}\big[S^{(N)}, \vec{T}^{(N)}\big]\delta^2\Big(\sum q_i - q\Big), \tag{9}$$

and their Fourier transform

$$I^{(N)}_{S,\vec{T}}(\vec{r}_1, \vec{r}_2) = \int\frac{d^2\vec{p}_1\,d^2\vec{p}_2}{(2\pi)^2}e^{i\vec{p}_1\vec{r}_1 + i\vec{p}_2\vec{r}_2}J^{(N)}_{S,\vec{T}}, \tag{10}$$

the recurrent relations can be written in the form

$$I^{(N)}_{S,\vec{T}}(\vec{r}_1, \vec{r}_2) = \pi I^{(N-1)}_{S,\vec{T}}(\vec{r}_1, \vec{r}_2)\Phi(\vec{r}_1, \vec{r}_2), \quad \Phi(\vec{r}_1, \vec{r}_2) = \frac{1}{\pi}\int\big(e^{i\vec{q}\vec{r}_2} - e^{i\vec{q}\vec{r}_1}\big)\frac{d^2q\,F(q^2)}{\vec{q}^{\,2}}. \tag{11}$$

In the Moliere approximation of atomic form factor in the Tomas–Fermi model (we use it below) the expression for form factor is [11]

$$\frac{F(q^2)}{\vec{q}^2} = \frac{1-F_A}{\vec{q}^2} = \sum_1^3 \frac{\alpha_i}{\mu_i^2+\vec{q}^2}, \tag{12}$$

with $\alpha_1 = 0.1$; $\alpha_2 = 0.55$; $\alpha_3 = 0.35$ and $\mu_i = (mZ^{1/3})b_i$ with $b_1 = 6.0$; $b_2 = 1.2$; $b_3 = 0.3$. In this case, the analytic expressions can be obtained

$$\Phi(\vec{r}_1,\vec{r}_2) = 2\sum_1^3 \alpha_i\big[K_0\big(\mu_i|r_2|\big) - K_0\big(\mu_i|r_1|\big)\big]. \tag{13}$$

For the pure Coulomb potential $F(q^2) = 1$, we have $\Phi^c(\vec{r}_1,\vec{r}_2) = \ln\frac{\vec{r}_1^2}{\vec{r}_2^2}$.

The boundary of recurrent relations is

$$I_S^{(1)}(\vec{r}_1,\vec{r}_2) = \frac{1}{2}K_0\big(m|\vec{r}_1-\vec{r}_2|\big)\Phi(\vec{r}_1,\vec{r}_2); \qquad \vec{I}_T^{(1)}(\vec{r}_1,\vec{r}_2) = \frac{im(\vec{r}_1-\vec{r}_2)}{2|\vec{r}_1-\vec{r}_2|}K_1\big(m|\vec{r}_1-\vec{r}_2|\big)\Phi(\vec{r}_1,\vec{r}_2), \tag{14}$$

with the $K_{0,1}(z)$-modified Bessel functions. The summation over the number of exchanged photons can be performed

$$J_S(\vec{p}_1,\vec{p}_2) = \frac{i}{2\nu}\int \frac{d^2r_1\,d^2r_2}{(2\pi)^2} e^{-i\vec{p}_1\vec{r}_1 - i\vec{p}_2\vec{r}_2} K_0\big(m|\vec{r}_1-\vec{r}_2|\big)\nu\big[e^{-i\nu\Phi(\vec{r}_1,\vec{r}_2)}-1\big];$$

$$\vec{J}_T(\vec{p}_1,\vec{p}_2) = \frac{-1}{2\nu}\int \frac{d^2r_1\,d^2r_2}{(2\pi)^2} e^{-i\vec{p}_1\vec{r}_1 - i\vec{p}_2\vec{r}_2} \frac{m(\vec{r}_1-\vec{r}_2)}{2|\vec{r}_1-\vec{r}_2|} K_1\big(m|\vec{r}_1-\vec{r}_2|\big)\big[e^{-i\nu\Phi(\vec{r}_1,\vec{r}_2)}-1\big]. \tag{15}$$

The differential cross section has the form

$$d\sigma = \frac{2\alpha\nu^2}{\pi^2}\big[|\vec{J}_T|^2 + m^2|J_S|^2 - 4x(1-x)\vec{J}_T\vec{\epsilon}\vec{J}_T\vec{\epsilon}^*\big]\,dx\,d^2p_1\,d^2p_2 = \frac{2\alpha\nu^2}{\pi^2}\big[W_{\text{unp}} + \xi_3 W_{\text{pol}}\cos(2\phi)\big]\,dx\,d^2p_1\,d^2p_2, \tag{16}$$

$$\epsilon_i\epsilon_j^* \to \frac{1}{2}[I + \xi_1\sigma_1 + \xi_3\sigma_3]_{ij}; \quad i,j = x,y, \tag{17}$$

where the polarization degree of photon, is described by means of $\xi_{1,3}$ the Stokes parameters, and ϕ is the angle between the vector $\vec{J}_T$ and the direction of maximal polarization of photon (if we choose the x axis along the direction of maximal polarization of photon, we put $\xi_1 = 0$; $P = \xi_3$), and

$$W_{\text{unp}} = \big[x^2 + (1-x)^2\big]|\vec{J}_T|^2 + m^2|J_S|^2; \qquad W_{\text{pol}} = -2x(1-x)|\vec{J}_T|^2. \tag{18}$$

For the screened potential (ignoring the experimental conditions of pair component definition) we use the expression for the phase given above. Performing the integration on pair momenta we obtain

$$2\pi\frac{d\sigma}{dx_1\,d\phi_1} = \frac{2\alpha}{m^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^\infty dy_1\int_0^\infty dy_2\,\big(1-\cos\big(\nu\phi_{12}^c\big)\big)\big[K_0^2(z) + \big[x_1^2+(1-x_1)^2\big]K_1^2(z) - 2x_1(1-x_1)\xi_3K_1^2(z)\cos(2\phi)\big],$$

$$z = \sqrt{y_1 + y_2 - 2\sqrt{y_1y_2}\cos\phi_0}, \tag{19}$$

where $\phi_{12}^c = \Phi^c(\vec{r}_2,\vec{r}_2) = (1/2)\ln(y_1/y_2)$. The azimuthal angle ϕ_0 is the angle between the direction of maximal photon polarization and the plane containing the direction of initial photon and electron (positron) from the pair.

In the case of pure Coulomb potential, integration in (19) diverges and must be regularized. We leave here this academic problem. For the screened potential we obtain:

$$2\pi\frac{d\sigma}{dx\,d\phi_1} = \frac{2\alpha}{m^2}\big[a(\nu) + \big(x^2+(1-x)^2\big)b(\nu) - 2x(1-x)\xi_3\cos(2\phi_1)b(\nu)\big]. \tag{20}$$

The ν dependence of the coefficients $a(\nu)$, $b(\nu)$ is shown in Fig. 1.

Further we will consider the realistic case of nonzero momentum transferred to nuclei $|\vec{q}|^2 \gg m^2$. For the pure Coulomb potential we have

$$\frac{d\sigma}{dx\,d\Omega_1\,d\Omega_2} = \frac{2\alpha\nu^2\omega^4}{\pi^2m^2}\frac{x^2(1-x)^2}{(\vec{q}^2)^2}\big|\Gamma(1-i\nu)\big|^4\big[W_u^c + \xi_3W_p^c\big], \tag{21}$$

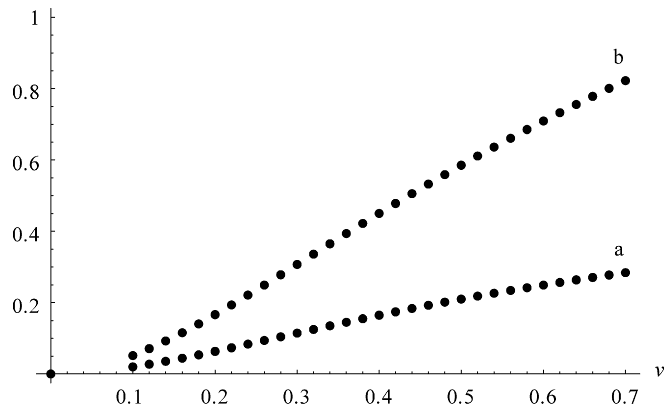

Fig. 1. The ν-dependence of coefficients a, b (see (19), (20)).

with

$$W_u^c = m^2\left[x^2+(1-x)^2\right]\left|(2F_1-F_2)\frac{\vec{p}_2}{c_2}+F_2\frac{\vec{p}_1}{c_1}\right|^2+\left|F_2-F_1+(2F_1-F_2)\frac{m^2}{c_2}-F_2\frac{m^2}{c_1}\right|^2;$$

$$W_p^c = -2x(1-x)m^2\left[|2F_1-F_2|^2\frac{\vec{p}_2^{\,2}}{c_2^2}\cos(2\phi_2)+|F_2|^2\frac{\vec{p}_1^{\,2}}{c_1^2}\cos(2\phi_1)+2\,\mathrm{Re}\left(F_2^*(2F_1-F_2)\right)\frac{|\vec{p}_2|}{c_2}\frac{|\vec{p}_1|}{c_1}\cos(\phi_1+\phi_2)\right], \quad (22)$$

and $F_1 = F(i\nu, -i\nu; 1; z)$; $F_2 = (1-i\nu)F(i\nu, 1-i\nu; 2; z)$ are the Gauss hypergeometric functions, $z = 1-\frac{m^2\vec{q}^{\,2}}{c_1c_2}$, the value of the transverse component of the pair is

$$|\vec{p}_1| = \omega x\theta_1; \qquad |\vec{p}_2| = \omega(1-x)\theta_2,$$

x, $1-x$ are the energy fractions of electron, positron. There $\phi_{1,2}$ are the angles between the direction of maximal polarization of photon and the transverse component of electron and the positron, $\theta_{1,2}$ are angles between the photon direction and lepton pair component emission, and $d\Omega_1\,d\Omega_2 = \theta_1\theta_2\,d\theta_1\,d\phi_1\,d\theta_2\,d\phi_2$ are phase volumes of the leptons.

In the case of small momentum transferred to nuclei $m^2 \ll \vec{q}^{\,2} \ll \vec{p}_1^{\,2} \approx \vec{p}_2^{\,2}$ we can put $z = 1$ in (22) and using $F_1 = F_2 = |\Gamma(1-i\nu)|^{-2}$, we reproduce the cross section in the Born approximation

$$\frac{d\sigma_B^c}{dx\,d\Omega_1\,d\Omega_2} = \frac{2\alpha^3Z^2\omega^4x^2(1-x)^2}{\pi^2(\vec{q}^{\,2})^2}\times\biggl[m^2\left(S^1\right)^2+\left(x^2+(1-x)^2\right)\left(\vec{T}^1\right)^2$$
$$-2x(1-x)\xi_3\left(\frac{\vec{p}_2^{\,2}}{c_2^2}\cos(2\phi_2)++\frac{\vec{p}_1^{\,2}}{c_1^2}\cos(2\phi_1)+\frac{|\vec{p}_2|}{c_2}\frac{|\vec{p}_1|}{c_1}\cos(\phi_1+\phi_2)\right)\biggr], \quad \vec{q}^{\,2} > m^2. \quad (23)$$

We note that the quantity in square brackets in the rhs of (23) is proportional to $\vec{q}^{\,2}$ at small $\vec{q}^{\,2}$. The experimental restrictions connected with pair component detection can be imposed as a domain of variation of energy fractions and angles of electron and positron.

For the large transversal momentum of the pair component $p_1 = p_2 = p \gg m$ we have

$$\frac{d\sigma_B}{d\Omega_1\,d\Omega_2\,dx} = \frac{\alpha^3\omega^4(x(1-x))^2}{2\pi^2p^6}\frac{1-2x(1-x)(1+\xi_3\cos(\phi_1+\phi_2))}{\cos^2(\frac{\phi_1-\phi_2}{2})},$$
$$\vec{q}^{\,2} = 4p^2\cos^2\left(\frac{\phi_1-\phi_2}{2}\right) \gg m^2. \quad (24)$$

2. The case of screened potential

In a more realistic case of electromagnetic field of atom described above, we do not succeed in obtaining the result in a closed form. So we calculate the values J_S, $\vec{J}_T$ by expansion in series up to the terms of the first order of ν.

After this expansion we have for the scalar structure

$$J_S(\vec{p}_1,\vec{p}_2) = J_S^{(1)}(\vec{p}_1,\vec{p}_2)+\nu J_S^{(2)}(\vec{p}_1,\vec{p}_2);$$
$$J_S^{(1)}(\vec{p}_1,\vec{p}_2) = \left(\frac{1}{c_1}-\frac{1}{c_2}\right)\sum_{i=1}^{3}\frac{\alpha_i}{q^2+\mu_1^2}; \qquad J_S^{(2)} = i\sum_{i,j=1}^{3}\alpha_i\alpha_j\left[\left(\frac{1}{c_1}+\frac{1}{c_2}\right)B-T_2-T_1\right] \quad (25)$$

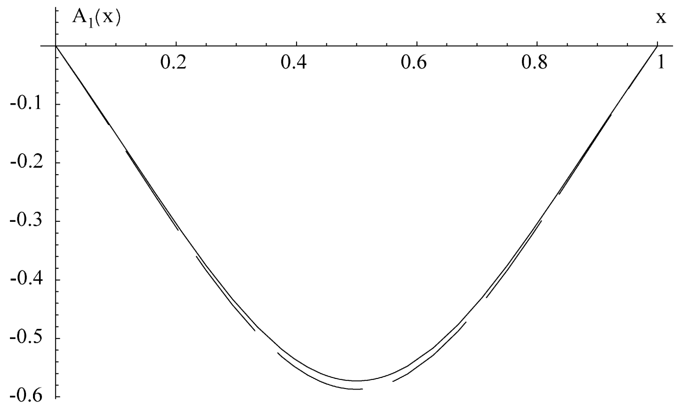

Fig. 2. Azimuthal asymmetry (see (20)); solid line $\nu = 0.3$, dashed line $\nu = 0.6$.

and an analogous expression for the vector structure is:

$$\vec{J}_T(\vec{p}_1, \vec{p}_2) = \vec{J}_T^{(1)}(\vec{p}_1, \vec{p}_2) + \nu \vec{J}_T^{(2)}(\vec{p}_1, \vec{p}_2);$$

$$\vec{J}_T^{(1)} = \left(\frac{\vec{p}_1}{c_1} + \frac{\vec{p}_2}{c_2}\right) \sum_{i,j=1}^{3} \frac{\alpha_i}{q^2 + \mu_1^2}; \qquad \vec{J}_T^{(2)} = i \sum_{i,j=1}^{3} \alpha_i \alpha_j \left[\left(\frac{\vec{p}_1}{a_1} + \frac{\vec{p}_2}{a_2}\right) B - \vec{p}_1 T_1 + \vec{p}_2 T_2 + \vec{T}_1 - \vec{T}_2\right]. \tag{26}$$

The quantities B, T_l, $\vec{T}_l$ are given in Appendix A.

The relevant contribution to the total cross section has the form

$$\frac{d\sigma}{dx\, d\Omega_1\, d\Omega_2} = \frac{d\sigma^{(1)}}{dx\, d\Omega_1\, d\Omega_2} + \frac{d\sigma^{(2)}}{dx\, d\Omega_1\, d\Omega_2}, \tag{27}$$

the first term is given above (see (23)) with the replacement $S^{(1)} \to J_s^{(1)}$; $\vec{T}^{(1)} \to \vec{J}_T^{(1)}$. By writing $\vec{J}^{(2)T}$ in the form $\vec{J}_T^{(2)} = A_1 \vec{p}_1 + A_2 \vec{p}_2$, the second term in (27) will be

$$\frac{d\sigma^{(2)}}{dx\, d\Omega_1\, d\Omega_2} = \frac{\alpha \nu^4}{4\pi^2} \big(x(1-x)\big)^2 \omega^4 I_2, \tag{28}$$

with

$$I_2 = m^2 \big(J_S^{(2)}\big)^2 + \big(x^2 + (1-x)^2\big)\big(\vec{J}_T^{(2)}\big)^2 - 2\xi_3 x(1-x)\big(A_1^2 \vec{p}_1^2 \cos(2\phi_1) + A_2^2 \vec{p}_2^2 \cos(2\phi_2)$$
$$+ 2A_1 A_2 |\vec{p}_1||\vec{p}_2| \cos(\phi_1 + \phi_2)\big). \tag{29}$$

High order contributions at $(Z\alpha)^n$, $n \gg 3$, can be expressed by an iteration procedure. However, the analytic expression turn out of the more complicated. Nevertheless, the formulae given here provide the accuracy of an order 1% for the case of pair photoproduction on light nuclei $Z\alpha < 0.3$.

3. Discussion

In famous papers of Bethe, Maximon and Olsen [2–4] the general theory of pair production and bremsstrahlung was built basing on the electron wave function in the Coulomb field. Part of these results was reproduced in a perturbation theory approach in [9]. Unfortunately, the expression for the differential cross sections which can be used in current experiments with specific cuts was rather poorly presented. It is the motivation of this Letter.

Our main results are presented in Eqs. (20), (21), (23), (27). These results are new and provide high accuracy since they are valid in all orders of PT. Using them the experimental restrictions can be put explicitly. In particular, the formula obtained above can describe the exclusive experiment with pair photoproduction on nuclei when both electron and positron are tagged.

In Fig. 2 the dependence of asymmetry defined as (see (20))

$$A(x, \xi_3) = \frac{d\sigma(\phi_1 = 0) - d\sigma(\phi_1 = \pi/2)}{d\sigma(\phi_1 = 0) + d\sigma(\phi_1 = \pi/2)} \tag{30}$$

is presented for $\xi_3 = 1$ as a function of electron energy fraction x and transversal momenta of pair components.

Asymmetry calculated using formula (20) turns out to be rather large compared to one obtained in [3] and reaches 40%. We should like to note that in [3] an emission restriction $\theta_\pm > 10^{-3}$ was put on. The main contribution, however, arises from small

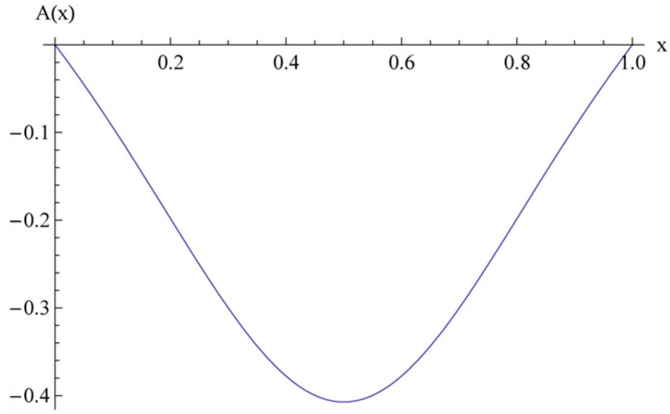

Fig. 3. Asymmetry in the Born approach; at fixed parameters: $p_1 = p_2 = 10$ MeV, $\xi_3 = 1$, $\phi_2 = \frac{\pi}{2}$.

values of $p_{1,2} \ll m$, which is implied in our formula (20). Our result can be used in an experimental set-up with the magnetic field.

Asymmetry

$$A(x, p_1, p_2, \phi_2, \xi_3) = \frac{d\sigma(\phi_1 = 0, x, p_1, p_2, \phi_2) - d\sigma(\phi_1 = \pi/2, x, p_1, p_2, \phi_2)}{d\sigma(\phi_1 = 0, x, p_1, p_2, \phi_2) + d\sigma(\phi_1 = \pi/2, x, p_1, p_2, \phi_2)}, \quad p_{1,2} = |\vec{p}_{1,2}|, \tag{31}$$

calculated by formulae with (23) at fix values p_1, p_2 is drawn in Fig. 3. The lowest order or correction to it for $\nu = 0.3$ is value of an order at 10% of the Born amplitude.

The differential distribution (23) is valid in the case of rather large values of transversal momenta of an electron–positron pair and provides the possibility to apply the experimental detection details.

The cross section in the case of unpolarized particles is of an order

$$\sigma \approx 170 \text{ mb}, \qquad Z = 79.$$

Accuracy of calculation is determined by the omitted terms

$$1 + O\left(\frac{\vec{p}^2}{s}, \frac{\alpha}{\pi} \ln \frac{\vec{p}^2}{m^2}\right). \tag{32}$$

The value of error is of an order at several per cent. The last term corresponds to pair component of final state interaction, which was not considered here.

This quantity does not depend on energy of the initial photon starting from its rather high values ($\omega > 50$ MeV) in accordance with the results of [13].

We show besides (see Appendix A) that the effects of higher orders of expansion on the parameter $\nu = Z\alpha$ disappear in the case when transversal components of electron and positron momentum considerably exceed the electron mass. The screening effects in asymmetry disappear in this limit as well.

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Appendix A

In the case of the screened potential the matrix element corresponding to two-photon exchange has the form

$$M^{(2)} = \frac{s(4\pi\alpha)^{5/2} Z^2}{4\pi} N_P \sum_{i,j=1}^{3} \alpha_i \alpha_j \bar{u}(p_2) \hat{R} \frac{\hat{P}}{s} v(p_1), \tag{A.1}$$

with

$$N_P = \frac{1}{s} \bar{u}(P') \hat{k} u(P), \quad \sum |N_P|^2 = 2, \tag{A.2}$$

and

$$\hat{R} = \int \frac{d^2k_1}{\pi} \frac{1}{\vec{k}_1^2 + \mu_i^2} \frac{1}{(\vec{q} - \vec{k}_1)^2 + \mu_j^2} [m\hat{e}S_2 + 2x\vec{T}_2\vec{e} + \hat{T}_2\hat{e}];$$

$$S_2 = \frac{1}{c_1} - \frac{1}{c_{1k}} + \frac{1}{c_2} - \frac{1}{c_{2k}}; \qquad \vec{T}_2 = \frac{\vec{p}_1}{c_1} + \frac{\vec{k}_1 - \vec{p}_1}{c_{1k}} - \frac{\vec{p}_2}{c_2} - \frac{\vec{k}_1 - \vec{p}_2}{c_{2k}}, \tag{A.3}$$

where c_l was defined above, $c_{lk} = (\vec{k}_1 - \vec{p}_l)^2 + m^2$.

We need to calculate the integrals

$$B = \int \frac{d^2k_1}{\pi(\vec{k}_1^2 + \mu_i^2)((\vec{q} - \vec{k}_1)^2 + \mu_j^2)}; \qquad (T_l, \vec{T}_l) = \int \frac{d^2k_1(1, \vec{k}_1)}{\pi(\vec{k}_1^2 + \mu_i^2)((\vec{q} - \vec{k}_1)^2 + \mu_j^2)((\vec{p}_l - \vec{k}_1)^2 + m^2)}. \tag{A.4}$$

Applying the Feynman joining procedure and performing the standard Feynman parameter integration we obtain

$$T_l = \int_0^1 dx \left\{ -\frac{2A_1 + B_1}{R\Delta} + \frac{2B_1}{R^{3/2}} L \right\}; \quad \Delta = A_1 + B_1 + C + 1,$$

$$\vec{T}_l = \int_0^1 dx\, \vec{p}_{xl} \left\{ -\frac{2C_1 + B_1}{R\Delta} + \frac{2C_1}{R^{3/2}} L \right\}; \quad L = \ln \frac{(B_1 + 2C_1 + \sqrt{R}\,)^2}{4C_1\Delta}, \; R = B_1^2 - 4A_1C_1 > 0, \tag{A.5}$$

with

$$A_1 = -\vec{p}_{xl}^2; \quad \vec{p}_{xl} = x\vec{q} + (1-x)\vec{p}_l;$$
$$\Delta = (1-x)m^2 + x\mu_j^2 + x(1-x)(q - p_l)^2; \qquad C_1 = \mu_i^2;$$
$$B_1 = (1-x)\big(\vec{p}_l^2 + m^2\big) + x\big(\vec{q}^2 + \mu_j^2\big) - \mu_i^2. \tag{A.6}$$

In the kinematic region $m^2 \sim \mu_i^2 \ll \vec{q}^2 \ll \vec{p}_i^2 = \vec{p}_2^2 = \vec{p}^2$ we obtain:

$$B = \frac{2}{\vec{q}^2} \ln \frac{\vec{q}^2}{\mu_i\mu_j}; \qquad T_1 = T_2 = \frac{2}{\vec{p}^2\vec{q}^2} \ln \frac{\vec{q}^2}{\mu_i\mu_j}; \qquad \vec{T}_1 = \vec{T}_2 = \frac{\vec{q}}{\vec{p}^2\vec{q}^2} \ln \frac{\vec{p}^2}{\mu_j^2}. \tag{A.7}$$

In this limit we have $S^{(2)} = \vec{T}^{(2)} = 0$. It can be shown that $S^{(n)} = \vec{T}^{(n)} = 0$, $n > 2$ is fulfilled in this limit as well.

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